Free energy versus internal energy potential for heavy quark systems at finite temperature

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(Dated: April 29, 2013)

Using the QCD sum rule with its operator product expansion reliably estimated from lattice calculations for the pressure and energy density of hot QCD matter, we calculate the strength of the J/ψ wave function at origin and find that it decreases with temperature when the temperature is above the transition temperature. This result is shown to follow exactly that obtained from the solution of the Schrödingier equation for a charm and anticharm quark pair using the free energy from lattice calculations as the potential and is in sharp contrast to that using the deeper potential associated with the internal energy, which shows an enhanced strength of the J/ψ wave function at origin. Our result thus has resolved the long-standing question of whether the free energy potential or the internal energy potential should be used in analyzing the spectrum of heavy quark systems at finite temperature.

PACS numbers: 14.40.Rt,24.10.Pa,25.75.Dw

arXiv:1304.4092v2 [nucl-th] 26 Apr 2013

Ever since the seminal work by Matsui and Satz [1], suggesting that the J/ψ suppression could be a signature for the formation of the quark-gluon plasma (QGP) in relativistic heavy ion collisions [2], there have been numerous theoretical as well as experimental studies from SPS to LHC on this subject. On the other hand, it is quite discomforting to know that even to this date, the question whether J/ψ dissolves immediately above the critical temperature T_c [3–5] or remains bounded at higher temperatures [6–8] is still not settled. The two scenarios have led to different phenomenological models based on either complete melting or sequential suppressions of charmonia in QGP for explaining the observed suppression of J/ψ production in relativistic heavy ion collisions, resulting thus in different conclusions about the phase transition in QCD and the properties of the quark-gluon plasma [9].

Given the fact that lattice calculations can calculate the heavy quark potential with great precision, it might seem that the question could be simply answered by just solving the Schrödinger equation and finding the ground state eigenvalues. However, it is not so clear whether one should use the free energy potential or the internal energy potential [10]. If one uses the free energy as the potential, the J/ψ then dissolves slightly above T_c whereas it remains bounded up to almost $2T_c$ if the internal energy is used. The difference between the two potentials can be traced to the question of whether one should subtract out the additional gluonic entropy contribution associated with adding the heavy quark pair in the QGP [11]. Because of this difference, the internal energy potential is deeper and also rises more sharply with increasing interquark distance, resulting in a J/ψ ground state wave function that is more localized at origin than that from the free energy potential. This effect leads to marked different behaviors in the temperature dependence of the J/ψ wave function at origin.

The J/ψ wave function at origin is the non-relativistic limit of the overlap of the J/ψ with the charm vector current and is thus a well defined gauge invariant and physically observable quantity. In this work, using QCD sum rules at finite temperature, we show that the temperature dependence of this overlap follows exactly the temperature dependence of the J/ψ wave function at origin that is obtained from using the free energy potential in the Schrödinger equation for the charm and anticharm quark pair. Unlike the uncertainty of the QCD sum rule approaches in predicting particular changes in the whole spectral density, the overlap can be obtained from the integrated part of the spectral density and thus reliably estimated. In fact, we will show that the zero temperature sum rule reproduces the strength needed to explain the dielectron partial width of the J/ψ and that the finite temperature calculations are equally reliable.

We start with the correlation function of the charm vector current $J_\mu = \bar{c}\gamma_\mu c$,

$$\Pi_{\mu\nu}(q) \;=\; i\int d^4x\, e^{iqx}\langle T[J_\mu(x)J_\nu(0)]\rangle. \qquad (1)$$

Inserting J/ψ as an intermediate state using the normalization $\langle J/\psi(p)|J/\psi(p)\rangle = (2\pi)^3 2\omega_p\delta^3(p-p)$, we have

$$\Pi^\mu_\mu(q) \;=\; -\frac{\langle J_\mu(0)|J/\psi\rangle\langle J/\psi|J^\mu(0)\rangle}{q^2-m^2_{J/\psi}}, \qquad (2)$$

where

$$\langle J_\mu(0)|J/\psi\rangle \;=\; i2\sqrt{m_{J/\psi}N_c}\epsilon_\mu\psi(0). \qquad (3)$$

In the above, ϵ_μ is the polarization vector and $\psi(0)$ reduces to the wave function of J/ψ at origin in the non-relativistic limit. Defining the charm quark pair polarization function $\Pi(q) = -\Pi^\mu_\mu(q)/(3q^2)$, we find the J/ψ contribution to its imaginary part to be

$$\frac{1}{\pi}\mathrm{Im}\Pi(s) = f_0\delta(s - m^2_{J/\psi}), \tag{4}$$

where

$$f_0 = \frac{12\pi}{m_{J/\psi}}|\psi(0)|^2. \tag{5}$$

This quantity is related to the dielectron decay width of J/ψ, $\Gamma^{e^+e^-}_{J/\psi} = \frac{16\pi\alpha^2 e_Q^2}{m^2_{J/\psi}}|\psi(0)|^2$. Using the empirical value $\Gamma^{e^+e^-}_{J/\psi} = 5.55$ keV for J/ψ in vacuum [12], we obtain the value

$$|\psi(0)| = 0.211 \pm 0.04 \text{ GeV}^{3/2} \tag{6}$$

for the overlap of the charm vector current with a free J/ψ or the wave function of a free $J//\psi$ at origin. We note that in identifying $\psi(0)$ as the J/ψ non-relativistic wave function at origin, Eq. (5) is expected to have several non-trivial corrections [13]. For example, within the non-relativistic limit, we could have used $2m_c$, with m_c being the charm quark mass, in the denominator, and this would amount to about 20% correction and hence the uncertainty given in Eq. (6). Since the relativistic corrections are expected to be temperature independent, we will study the temperature dependence of the overlap of charm vector current with J/ψ at finite temperature relative to its vacuum value.

The residue of the charmonium correlator f_0 given in Eq. (5) can be reliably calculated in the QCD sum rule method [14, 15]. While masses of heavy quark systems have been mostly studied in the moment sum rule, we use here the Borel transformed sum rule, which suppresses the contribution from the continuum, to calculate f_0 as it is more robust under changes of the continuum [16]. The generalization of the Borel sum rule to finite temperature is well founded as the operator product expansion (OPE) is well known up to dimension 4 operators, which are needed for the present analysis, by using the lattice data on the energy density and pressure of hot QCD matter. The method has been used to study the masses of J/ψ at finite density [17] as well as at finite temperature [18]. Recently, the method has been combined with the maximum entropy method to reconstruct the charmonium spectral density at finite temperature [4]. The Borel sum rule for f_0 only involves the strength at the pole, which is an integrated quantity, and thus does not depend on the details of the spectral density. Hence, we concentrate on the temperature dependence of f_0.

Specifically, the OPE for the charm quark pair polarization function $\Pi(q)$ is equated to the spectral density via the following dispersion relation after Borel transformation:

$$\mathcal{M}(M^2) = \int_0^\infty ds e^{-s/M^2}\mathrm{Im}\Pi(s), \tag{7}$$

after neglecting the thermal factor $\tanh[s/2T]$ in the spectral density as its correction to charmonium or the continuum is negligible up to temperatures of $1.1T_c$ [18, 19]. For the spectral density, we assume the following form:

$$\mathrm{Im}\Pi(s) = \mathrm{Im}\Pi^{J/\psi}(s) + \theta(s - s_0)\mathrm{Im}\Pi^{\mathrm{pert}}(s) \tag{8}$$

with the first term given by the pole form in Eq. (4).

The Borel transformed OPE and its temperature dependence up to dimension 4 operators are:

$$\begin{aligned}\mathcal{M}(M^2) &= e^{-\nu}\pi A(\nu)[1 + \alpha_s(M^2)a(\nu) + b(\nu)\phi_b(T) \\ &\quad + c(\nu)\phi_c(T)]\end{aligned} \tag{9}$$

with $\nu = 4m_c^2/M^2$ being a dimensionless scale parameter. The Wilson coefficients $a(\nu)$, $b(\nu)$, and $c(\nu)$ are summarized in Ref. [20], where the temperature dependence of the scalar and twist-2 gluon condensates ϕ_b and ϕ_c are also given. We note that the truncation of the OPE up to dimension 4 operators is valid up to temperature of $T = 1.1T_c$ as well [19].

Using the OPE side of Eq.(9), we can express the residue f_0 as

$$f_0 = e^{m^2_{J/\psi}(M^2)/M^2}[\mathcal{M}^{\mathrm{OPE}}(M^2) - \mathcal{M}^{\mathrm{cont}}(M^2; s_0)], \tag{10}$$

where the J/ψ mass is given by

$$m^2_{J/\psi} = -\frac{\frac{\partial}{\partial(1/M^2)}(\mathcal{M}^{\mathrm{OPE}}(M^2) - \mathcal{M}^{\mathrm{cont}}(M^2; s_0))}{\mathcal{M}^{\mathrm{OPE}}(M^2) - \mathcal{M}^{\mathrm{cont}}(M^2; s_0)}. \tag{11}$$

The continuum part of the correlator $\mathcal{M}^{\mathrm{cont}}$ depends on the threshold parameter s_0. Following the method used in Ref. [21] for the estimation of $m_{J/\psi}$, we determine s_0 by requiring $|\psi(0)| = m_{J/\psi}f_0/(12\pi)$ to be least sensitive to M^2. We have confirmed that the M^2 dependence of $|\psi(0)|$ is quite similar to that of $m_{J/\psi}$, but the resultant value of s_0 differs slightly from the case in which the Borel curve for $m_{J/\psi}$ is optimized. We have also estimated the systematic error due to the M^2 dependence of $|\psi(0)|$ and found it to be at most 0.001 GeV$^{3/2}$. We have further examined the effects of finite width above T_c. Introducing the width, however, brings an ambiguity on the stabilization of the Borel curve due to possible absence of a solution [20]. Nevertheless, the change due to the introduction of nonzero width can be understood as a result of the strong correlation among the mass, the width and the threshold as shown in Ref. [20]. Since a nonzero width leads to a larger mass while keeping the residue unchanged, the resultant $|\psi(0)|$ becomes slightly

larger as shown in Fig. 1 by the Borel curves for $|\psi(0)|$ at $T = 1.0\ T_c$ for three different values of the J/ψ width. Nevertheless, the difference is small and our prediction for $|\psi(0)|$ is thus robust.

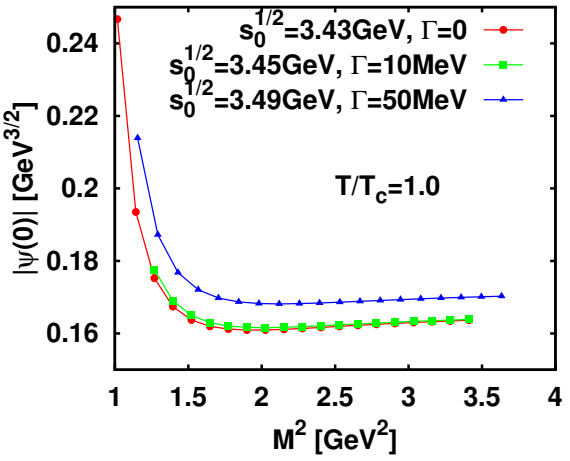

FIG. 1: (Color online) Borel curves for $|\psi(0)|$ at $T = 1.0\ T_c$ for different values of J/ψ width and continuum threshold.

Figure 2 shows the temperature dependence of $|\psi(0)|$ and $m_{J/\psi}$ obtained by the Borel stability analysis at each temperature up to slightly above $T = 1.05\ T_c$ beyond which the OPE becomes less reliable [18]. Irrespective of the J/ψ width used in the analysis, both $|\psi(0)|$ (filled symbols, left vertical axis) and $m_{J/\psi}$ (open symbols, right vertical axis) are seen to decrease with increasing temperature.

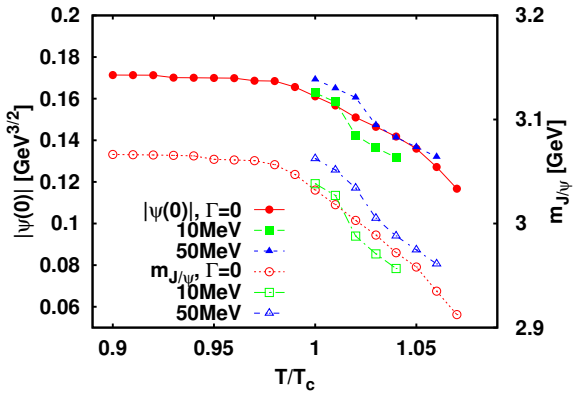

FIG. 2: (Color online) Temperature dependence of $|\psi(0)|$ (filled symbols, left vertical axis) and mass of J/ψ (open symbols, right vertical axis) obtained from the QCD sum rule for different values of J/ψ width.

To find out which potential between a charm and anticharm quark pair correctly reproduces $|\psi(0)|$ obtained from the QCD sum rule, we solve the following Schrödinger equation between a charm and anticharm pair:

$$\left[2m_c - \frac{1}{m_c}\nabla^2 + V(r,T)\right]\psi(r,T) = M\psi(r,T), \quad (12)$$

where $m_c = 1.25$ GeV is the bare mass of charm quark and $\psi(r,T)$ is the charmonium wave function at temperature T. Introducing the potential energy at infinitely large distance, $V(r=\infty,T)$, the Schrödinger equation is modified to [22, 23]

$$\left[-\frac{\nabla^2}{m_c} + \widetilde{V}(r,T)\right]\psi(r,T) = -\varepsilon\psi(r,T), \quad (13)$$

where $\widetilde{V}(r,T) \equiv V(r,T) - V(r=\infty,T)$, and it vanishes at infinitely large distance, and $\varepsilon = 2m_c + V(r=\infty,T) - M$ is the binding energy of J/ψ at temperature T. For the heavy quark potential, we use either the free energy between a heavy quark-antiquark pair that is extracted from lattice calculations [10, 23, 24] or the more attractive one based on the internal energy by adding the contribution from the entropy density.

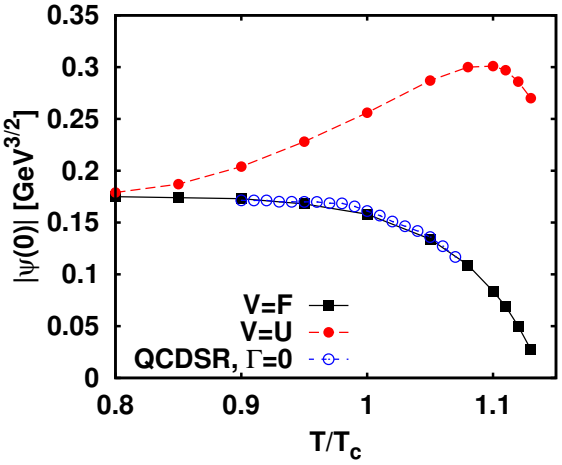

FIG. 3: (Color online) Temperature dependence of $|\psi(0)|$ obtained from the free energy (filled squares) and internal energy (filled circles) potentials together with that from the QCD sum rule (open circles).

Figure 3 shows the temperature dependence of the strength of the J/ψ wave function at the origin obtained by solving the Schrödinger with the two different potentials, together with the result from the QCD sum rule in the case of vanishing width. With the internal energy as the potential, the strength is seen to increase by almost a factor of two at slightly above the critical temperature. On the other hand, the strength decreases monotonically with temperature when the free energy is used as the potential, strikingly similar to the result from QCD sum rules in both its behavior and values. Even allowing for

$\pm 20\%$ uncertainty in the sum rule result, the case with internal energy is outside the range of expectations from sum rule calculations. Hence we can conclude that to correctly reproduce the non-relativistic wave function of J/ψ at finite temperature, one should use the free energy potential in the Schrödinger equation.

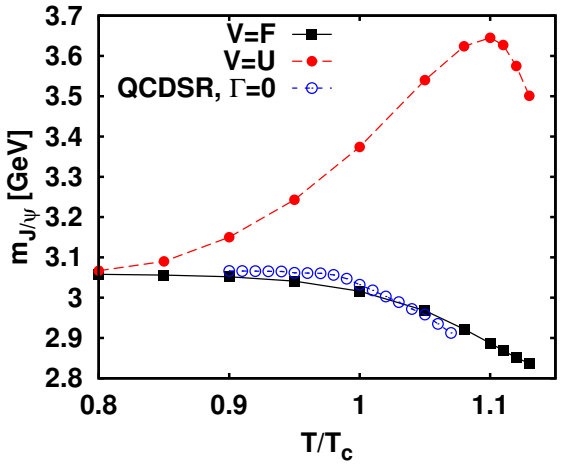

FIG. 4: (Color online) Temperature dependence of $m_{J/\psi}$ obtained from solving the Schrödinger equation using either the free energy (filled squares) or the internal energy (filled circles) and from the QCD sum rule analysis (open circles).

In Fig. 4, we plot the mass of J/ψ obtained from Eq. (12) with different potentials, and compare the results to that obtained from the sum rule analysis of Eq. (11). Again, we find that the result obtained from using the free energy is consistent with that from the QCD sum rule analysis.

The result obtained in the present study that the potential between a heavy quark-antiquark pair in QGP is determined by their free energy has important phenomenological consequences. For J/ψ, this leads to a dissociation temperature that is only slightly above T_c [25]. As a result, all charmonium states would essentially melt in the quark-gluon plasma. The observed J/ψ's in relativistic heavy ion collisions are then either from initial hard collisions that occur outside the QGP or from the recombination of charm and anticharm quarks in the QGP during hadronization.

It should be noted, however, that in obtaining our result from the Schrödinger equation, we have kept the charm quark mass to its zero temperature value and also taken the heavy quark potential to be real. These quantities taken separately are gauge dependent, and different prescriptions can be used to determine their values at finite temperature. Since the heavy quark potential is expected to acquire a gauge dependent imaginary part at finite temperature [26, 27], a stronger potential and a different charm quark in-medium mass may then be needed to reproduce gauge invariant and physically observable quantities like $|\psi(0)|$, $m_{J/\psi}$ and the J/ψ dissociation temperature.

Acknowledgements We thank useful comments from A. Rothkopf and Y. Akamatsu. This work was supported in part by the Korean Research Foundation under Grant Nos. KRF-2011-0020333 and KRF-2011-0030621, the Yukawa International Program for Quark-Hadron Sciences at Kyoto University and the Grant-in-Aid for Scientific Research from JSPS No. 24540271 as well as the US National Science Foundation under Grant No. PHY-1068572, the US Department of Energy under Contract No. DE-FG02-10ER41682, and the Welch Foundation under Grant No. A-1358.

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